Mesh Statistics for Robust Curvature Estimation

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The stage

Surface curvature

- κ_1 and κ_2
- κ_G and κ_H
- local differential properties of smooth surfaces

Triangle mesh

- represents the surface
- connectivity
- geometry

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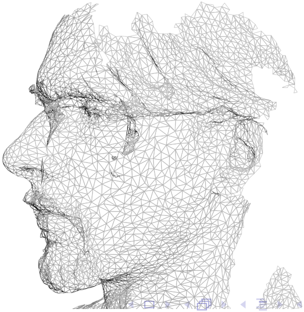

The problem

Objective

Estimate curvature at vertices of the triangle mesh

in what sense?

- curvature of the approximated surface
- cannot be (really) done

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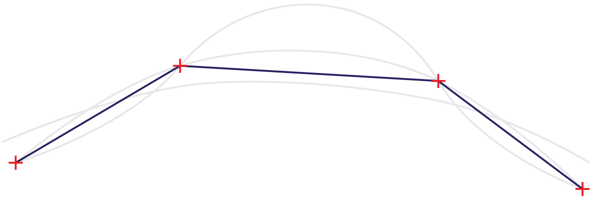

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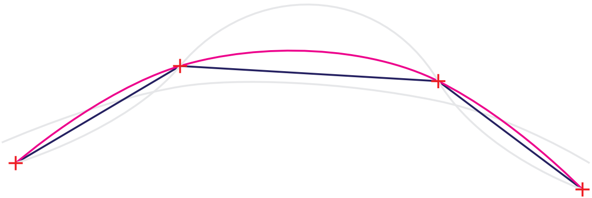

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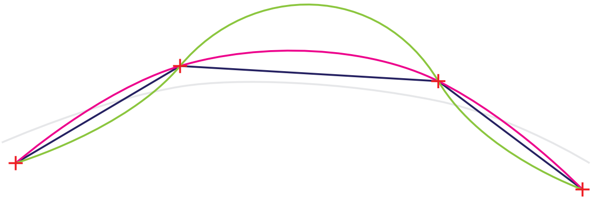

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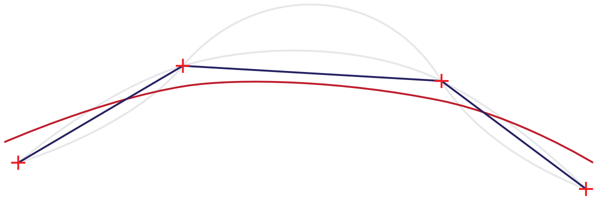

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Many approaches exist

- fitting a smooth surface
- integrating over a finite area
- estimating shape operator
- ...

Key question

Which estimator should I use?

Depends on the character of the data.

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The experiment

- generate many smooth surfaces (curvature known)
- generate many meshes
  - different properties
  - each mesh homogeneous
- compute exact curvatures
- test many estimators

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Can we guess which estimator will work well just from the mesh properties?

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Experiment configuration


```
<TestConfig  xmlns="http://Zcu.CurvatureBenchmark/testconfig.xsd">
  <Sources>
    <SphereSource MinRadius="1" MaxRadius="10" Subdivision="4" />
  </Sources>
  <Distorters>
    <NoDistorter/>
    <NormalsMax/>
  </Distorters>
  <Estimators>
    <RusinkiewiczEstimator Active="true" />
    <GoldfeatherInterranteEstimator Active="false" method="1" neighborhoo
    <CSMEstimator Active="true" RangeKEdge="2" />
  </Estimators>
  <Evaluators>
    <BasicEvaluator />
  </Evaluators>
</TestConfig>
```

Data Sources

- explicit functions $z = f(x, y)$
- implicit functions $f(x, y, z) = 0$;
- NURBS surfaces

Explicit functions

- function $z(x, y) = Ax^2 + By^2 + Cxy + Dx + Ey$
  - A, B, C, D, E random
  - different (differentiable) functions can be easily plugged in
- different samplings
  - regular rectangular
  - regular triangular
  - randomized (regular triangular + noise)
  - random
- different densities

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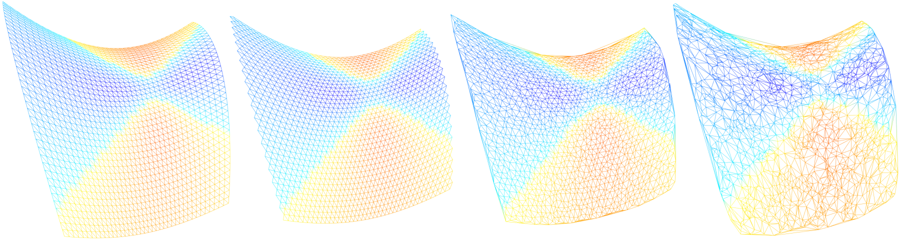

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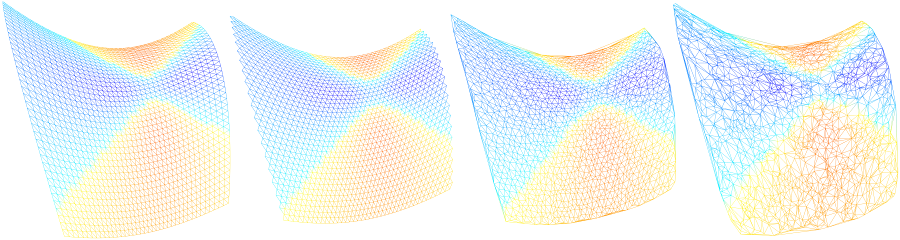

Implicit functions

- function $Asin(Bx) + Csin(Dy) + sin(Ez) = 0$

- different triangulations
  - Marching cubes
  - Adaptive Dual Contouring
- different evaluation
  - inexact (interpolation on grid edges)
  - exact (interval subdivision on grid edges)

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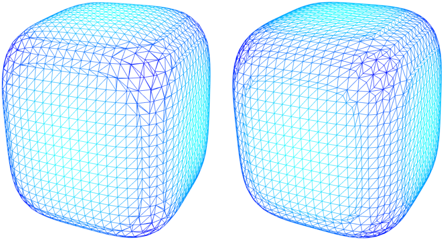

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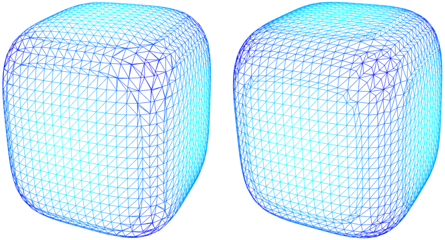
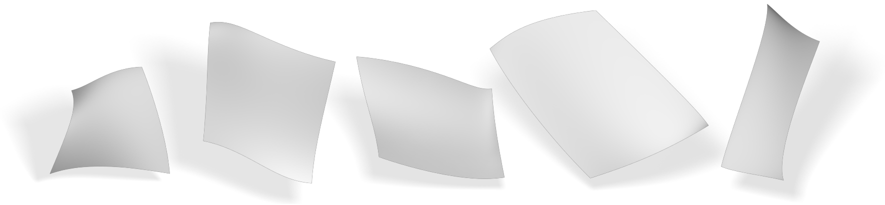


NURBS surfaces

- commonly used parametric surfaces
- cubic surfaces used
- same sampling schemes in UV as with explicit surfaces in XY
- random positions of control mesh
- random weights



Distortions

- Gaussian noise
  - common scanning artifact
  - different strengths
- uniform noise
  - common compression artifact
  - different strengths
- computation of vertex normals
  - needed by some estimators
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Estimators

- [Meyer] (cotan discretization of Laplacian, angle deficit)
- [Goldfeather and Interrante]
  - circe fitting
  - parabola fitting
- [Rusinkiewicz] (estimation of II)
- [Kalogerakis et al.] (adaptive estimation of II)
- [Cohen-Steiner and Morvan] (shape operator estimation)
- [Zhihong et al.] (Bézier patches)
- [Fünfzig et al.] (PNG-1 patches)
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Note

We do not discuss whether using those is appropriate, we just measure the results

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Evaluation

Absolute error for i-th estimator and j-th mesh:

$$e_i(\mathcal{M}_j) = \frac{1}{N}\sum_{k=0}^{N-1}(\|\kappa_1^k - \hat{\kappa_1^k}\| + \|\kappa_2^k - \hat{\kappa_2^k}\|)$$

Relative error, related to the best achieved accuracy:

$$\hat{e}_i(\mathcal{M}_j) = \frac{e_i(\mathcal{M}_j) - e_{min}(\mathcal{M}_j)}{e_{min}(\mathcal{M}_j)}, e_{min}(\mathcal{M}_j) = \min_i(e_i(\mathcal{M}_j))$$

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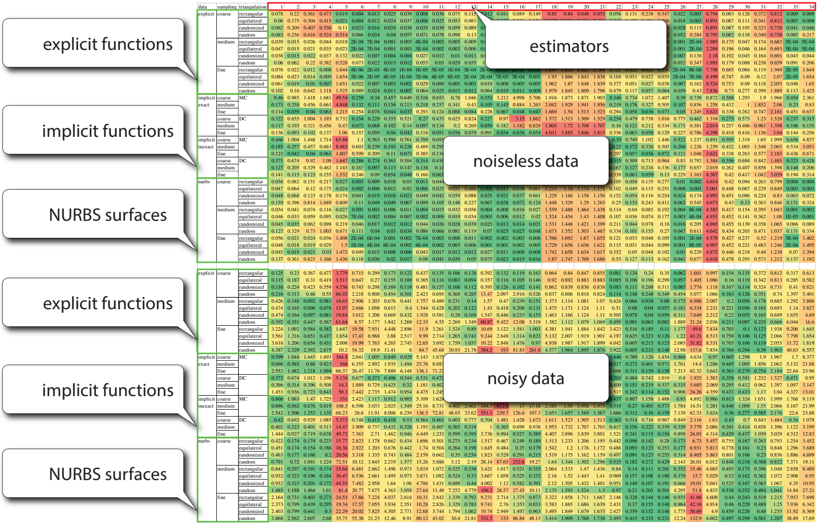
Results

data	sampling	triangulation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
explicit	coarse	rectangular	0.078	0.22	0.361	0.473	0.019	0.004	0.013	0.025	0.039	0.008	0.036	0.075	0.115	0.012	0.041	0.089	0.149	0.85	0.84	0.848	0.855	0.056	0.131	0.238	0.347	0.022	0.003	0.794	0.083	0.128	0.372	0.812	0.009	0.009
		equilateral	0.06	0.175	0.304	0.415	0.021	0.004	0.012	0.024	0.037	0.006	0.025	0.053	0.081	0.01	0.037	0.081	0.136	0.897	0.889	0.884	0.884	0.048	0.102	0.194	0.296	0.018	0.003	0.891	0.087	0.111	0.341	0.812	0.007	0.008
		randomized	0.062	0.209	0.407	0.536	0.11	0.023	0.016	0.024	0.038	0.035	0.039	0.059	0.089	0.019	0.035	0.071	0.117	0.85	0.84	0.837	0.837	0.056	0.108	0.206	0.309	0.028	0.113	0.891	0.087	0.105	0.323	0.728	0.041	0.046
		random	0.083	0.236	0.416	0.524	0.514	0.066	0.034	0.04	0.057	0.071	0.078	0.098	0.13	0.044	0.048	0.1	0.172	0.831	0.805	0.812	0.819	0.081	0.127	0.229	0.332	0.052	0.284	0.795	0.082	0.118	0.349	0.738	0.085	0.217
	medium	rectangular	0.039	0.015	0.026	0.044	0.018	2E-04	7E-04	0.001	0.003	4E-04	0.002	0.005	0.009	0.002	0.002	0.005	0.008	1.143	1.086	1.075	1.07	0.075	0.018	0.025	0.054	0.001	2E-04	1.985	0.175	0.047	0.174	0.683	5E-04	5E-04
		equilateral	0.047	0.015	0.021	0.035	0.023	2E-04	7E-04	0.001	0.003	3E-04	0.002	0.003	0.006	0.001	0.002	0.004	0.008	1.179	1.13	1.114	1.106	0.069	0.018	0.022	0.05	0.001	2E-04	2.204	0.196	0.046	0.164	0.693	5E-04	5E-04
		randomized	0.038	0.015	0.022	0.037	0.132	0.022	0.007	0.004	0.004	0.027	0.012	0.01	0.011	0.017	0.008	0.007	0.009	1.196	1.133	1.115	1.107	0.112	0.023	0.023	0.052	0.007	0.138	2.18	0.192	0.045	0.164	0.691	0.043	0.044
		random	0.06	0.062	0.22	0.382	0.528	0.073	0.022	0.013	0.012	0.055	0.03	0.029	0.035	0.05	0.015	0.011	0.014	1.176	1.091	1.068	1.057	0.167	0.042	0.04	0.067	0.032	0.347	1.993	0.179	0.088	0.238	0.659	0.091	0.206
	fine	rectangular	0.076	0.022	0.012	0.008	1.644	4E-06	2E-05	4E-05	1E-04	1E-05	5E-05	1E-04	3E-04	2E-04	1E-04	2E-04	9E-04	1.899	1.824	1.803	1.799	0.121	0.033	0.024	0.036	2E-04	8E-06	7.798	0.685	0.084	0.119	1.949	2E-05	1.644
		equilateral	0.084	0.023	0.014	0.009	1.654	5E-06	2E-05	4E-05	1E-04	7E-06	4E-05	8E-05	2E-04	2E-04	7E-05	3E-04	0.001	1.93	1.866	1.843	1.838	0.104	0.031	0.022	0.035	2E-04	7E-06	8.499	0.747	0.09	0.12	2.07	2E-05	1.654
		randomized	0.061	0.019	0.01	0.007	1.651	0.022	0.007	0.003	0.002	0.029	0.009	0.005	0.003	0.019	0.008	0.005	0.005	1.962	1.87	1.848	1.839	0.377	0.051	0.027	0.038	0.007	0.162	8.524	0.753	0.09	0.118	2.055	0.048	1.65
		random	0.102	0.16	0.642	1.118	1.525	0.089	0.024	0.011	0.007	0.064	0.025	0.015	0.012	0.064	0.019	0.011	0.009	1.979	1.845	1.809	1.796	0.479	0.117	0.057	0.064	0.039	0.43	7.836	0.73	0.277	0.399	1.889	0.113	1.425
implicit exact	coarse	MC	0.46	0.985	1.418	1.681	49.54	0.259	0.34	0.457	0.649	0.516	0.653	0.78	1.046	0.353	3.212	4.998	5.706	1.916	1.873	1.871	1.903	0.346	0.734	1.072	1.407	0.39	0.739	0.872	0.588	1.293	1.9	1.966	0.454	2.361
	medium		0.173	0.258	0.456	0.661	4.848	0.132	0.111	0.156	0.213	0.218	0.257	0.341	0.43	0.105	0.145	0.884	1.263	2.042	1.929	1.941	1.956	0.219	0.176	0.327	0.505	0.107	0.856	1.258	0.432	1	1.852	2.06	0.23	0.83
	fine		0.114	0.039	0.04	0.061	1.215	0.254	0.078	0.044	0.037	0.293	0.124	0.088	0.084	0.228	0.067	0.048	0.047	1.669	1.54	1.513	1.523	0.294	0.058	0.036	0.073	0.05	1.245	3.621	0.338	0.262	0.747	2.181	0.451	0.637
	coarse	DC	0.322	0.655	1.004	1.193	0.732	0.154	0.229	0.355	0.521	0.27	0.433	0.625	0.824	0.227	0.97	3.15	1.862	1.572	1.513	1.509	1.529	0.254	0.479	0.738	1.016	0.775	0.462	1.318	0.275	0.575	1.23	1.528	0.237	0.315
	medium		0.117	0.193	0.321	0.456	0.47	0.072	0.068	0.102	0.14	0.097	0.134	0.2	0.269	0.059	0.182	1.182	0.829	1.805	1.72	1.708	1.707	0.16	0.121	0.212	0.334	0.171	0.381	2.014	0.257	0.406	0.961	1.398	0.106	0.135
	fine		0.136	0.093	0.102	0.137	1.06	0.157	0.059	0.04	0.042	0.116	0.051	0.056	0.079	0.091	0.034	0.036	0.054	4.031	3.885	3.846	3.815	0.336	0.063	0.058	0.129	0.227	0.786	4.298	0.418	0.416	1.136	3.04	0.144	0.256
implicit inexact	coarse	MC	0.466	1.004	1.448	1.714	63.88	1.1	0.563	0.594	0.761	0.709	0.695	0.836	1.1	0.737	3.06	5.026	5.264	1.927	1.897	1.9	1.935	0.33	0.749	1.102	1.446	0.522	1.237	0.891	0.595	1.319	1.65	1.999	0.658	4.837
	medium		0.185	0.257	0.457	0.663	8.483	0.603	0.239	0.193	0.226	0.489	0.295	0.351	0.435	0.324	0.188	0.457	4.406	2.048	1.929	1.942	1.958	0.22	0.172	0.326	0.505	0.206	1.226	1.259	0.432	1.003	1.548	2.065	0.534	3.051
	fine		0.123	0.042	0.04	0.061	1.807	0.599	0.209	0.11	0.073	0.385	0.134	0.083	0.077	0.265	0.075	0.047	0.043	1.674	1.54	1.513	1.522	0.297	0.057	0.036	0.072	0.121	1.048	3.621	0.338	0.263	0.577	2.183	0.436	0.671
	coarse	DC	0.371	0.674	0.92	1.08	1.647	0.286	0.274	0.363	0.504	0.314	0.416	0.589	0.769	0.383	1.29	2.085	1.313	1.595	1.521	1.504	1.512	0.315	0.509	0.713	0.964	0.83	0.792	1.384	0.356	0.694	0.842	1.485	0.323	0.428
	medium		0.127	0.205	0.329	0.462	1.143	0.167	0.097	0.113	0.147	0.138	0.14	0.201	0.269	0.103	0.147	0.617	8.703	1.811	1.722	1.708	1.707	0.167	0.127	0.216	0.336	0.177	0.657	2.019	0.262	0.407	0.858	1.398	0.148	0.206
	fine		0.141	0.115	0.123	0.155	1.532	0.246	0.09	0.054	0.048	0.166	0.061	0.059	0.08	0.128	0.042	0.04	0.056	4.032	3.884	3.846	3.815	0.339	0.063	0.059	0.13	0.229	1.163	4.307	0.42	0.417	1.042	3.039	0.194	0.314
nurbs	coarse	rectangular	0.056	0.082	0.151	0.217	0.027	0.003	0.009	0.018	0.03	0.011	0.044	0.09	0.146	0.008	0.023	0.047	0.081	1.261	1.212	1.206	1.198	0.105	0.058	0.135	0.277	0.01	0.002	4.614	0.42	0.094	0.263	0.799	0.004	0.004
		equilateral	0.047	0.064	0.12	0.175	0.024	0.002	0.006	0.012	0.02	0.006	0.025	0.051	0.086	0.006	0.017	0.036	0.061	1.233	1.18	1.174	1.171	0.102	0.049	0.115	0.251	0.008	0.001	5.001	0.448	0.087	0.229	0.849	0.003	0.003
		randomized	0.048	0.068	0.123	0.178	0.174	0.041	0.019	0.018	0.023	0.049	0.042	0.059	0.088	0.025	0.021	0.037	0.061	1.229	1.166	1.158	1.156	0.152	0.054	0.116	0.254	0.024	0.174	4.993	0.451	0.086	0.224	0.83	0.063	0.072
		random	0.159	0.396	0.814	1.049	0.809	0.11	0.049	0.049	0.067	0.099	0.105	0.148	0.227	0.067	0.038	0.072	0.124	1.448	1.329	1.29	1.265	0.25	0.133	0.243	0.411	0.062	0.545	4.673	0.43	0.13	0.363	0.846	0.131	0.324
	medium	rectangular	0.054	0.041	0.074	0.116	0.027	0.001	0.003	0.006	0.011	0.004	0.015	0.032	0.056	0.004	0.008	0.016	0.027	1.559	1.488	1.464	1.438	0.114	0.04	0.085	0.192	0.004	8E-04	4.583	0.417	0.154	0.395	1.043	0.001	0.002
		equilateral	0.046	0.033	0.059	0.095	0.026	7E-04	0.002	0.004	0.007	0.002	0.009	0.019	0.034	0.003	0.006	0.012	0.02	1.524	1.454	1.43	1.408	0.107	0.036	0.074	0.177	0.003	4E-04	4.951	0.452	0.141	0.362	1.08	1E-03	0.001
		randomized	0.045	0.035	0.062	0.098	0.219	0.046	0.017	0.012	0.012	0.044	0.026	0.028	0.039	0.025	0.013	0.014	0.021	1.531	1.448	1.421	1.399	0.211	0.044	0.078	0.18	0.016	0.205	4.965	0.455	0.139	0.358	1.065	0.066	0.089
		random	0.127	0.329	0.73	1.003	0.671	0.111	0.04	0.03	0.034	0.086	0.066	0.081	0.119	0.07	0.025	0.027	0.048	1.673	1.552	1.503	1.467	0.334	0.101	0.155	0.27	0.047	0.611	4.642	0.434	0.203	0.471	1.037	0.131	0.334
	fine	rectangular	0.056	0.021	0.024	0.036	1.408	2E-04	6E-04	0.001	0.002	7E-04	0.003	0.006	0.011	0.002	0.002	0.003	0.006	1.766	1.692	1.67	1.655	0.121	0.033	0.048	0.105	0.001	1E-04	4.578	0.427	0.237	0.52	1.219	3E-04	1.402
		equilateral	0.048	0.018	0.019	0.029	1.5	1E-04	4E-04	8E-04	0.002	4E-04	0.002	0.003	0.006	0.001	0.001	0.002	0.005	1.729	1.656	1.636	1.621	0.115	0.031	0.044	0.099	0.001	8E-05	4.907	0.452	0.221	0.483	1.246	2E-04	1.495
		randomized	0.045	0.019	0.021	0.03	1.472	0.049	0.015	0.008	0.006	0.043	0.017	0.012	0.012	0.027	0.011	0.008	0.008	1.742	1.658	1.634	1.617	0.352	0.05	0.048	0.102	0.01	0.229	4.872	0.446	0.218	0.48	1.238	0.07	1.394
		random	0.137	0.361	0.823	1.166	1.436	0.118	0.036	0.02	0.017	0.08	0.041	0.039	0.05	0.075	0.023	0.015	0.016	1.87	1.747	1.709	1.686	0.53	0.127	0.113	0.162	0.041	0.677	4.618	0.478	0.293	0.573	1.212	0.137	1.192

explicit	coarse	rectangular	0.125	0.23	0.367	0.477	3.779	0.715	0.299	0.173	0.122	0.437	0.135	0.106	0.128	0.392	0.132	0.119	0.163	0.864	0.84	0.847	0.855	0.081	0.134	0.24	0.35	0.062	1.601	0.997	0.154	0.135	0.372	0.812	0.317	0.613
		equilateral	0.115	0.187	0.31	0.419	3.517	0.647	0.27	0.155	0.109	0.385	0.116	0.083	0.094	0.357	0.116	0.105	0.146	0.92	0.892	0.883	0.883	0.085	0.106	0.196	0.299	0.057	1.405	1.086	0.16	0.119	0.342	0.813	0.285	0.582
		randomized	0.116	0.224	0.423	0.559	4.536	0.743	0.298	0.169	0.118	0.481	0.157	0.106	0.112	0.395	0.126	0.102	0.141	0.862	0.839	0.836	0.838	0.083	0.111	0.208	0.311	0.085	1.774	1.118	0.167	0.114	0.324	0.731	0.41	0.821
		random	0.216	0.313	0.46	0.55	66.92	2.218	0.808	0.454	0.301	2.423	0.695	0.368	0.285	13.47	2.097	2.916	0.526	0.837	0.806	0.818	0.824	0.114	0.148	0.248	0.349	0.454	5.077	1.066	0.163	0.126	0.351	0.74	1.397	5.401
	medium	rectangular	0.426	0.148	0.092	0.081	16.63	2.906	1.203	0.676	0.441	1.757	0.489	0.231	0.14	1.57	0.47	0.239	0.151	1.373	1.114	1.081	1.07	0.424	0.066	0.038	0.06	0.175	6.986	2.007	0.2	0.056	0.178	0.685	1.292	3.806
		equilateral	0.474	0.165	0.096	0.078	15.97	2.666	1.098	0.615	0.4	1.544	0.428	0.202	0.122	1.43	0.418	0.208	0.131	1.475	1.171	1.124	1.11	0.51	0.08	0.04	0.057	0.181	6.334	2.231	0.221	0.056	0.165	0.693	1.14	3.827
		randomized	0.474	0.164	0.097	0.081	19.84	3.012	1.206	0.669	0.432	1.928	0.581	0.28	0.169	1.547	0.446	0.233	0.153	1.463	1.166	1.124	1.11	0.505	0.078	0.04	0.059	0.311	7.649	2.212	0.22	0.055	0.165	0.689	1.655	4.69
		random	0.592	0.351	0.447	0.567	61.64	8.57	3.177	1.842	1.269	12.33	4.55	2.269	1.349	60.87	9.422	12.08	5.319	1.382	1.112	1.079	1.069	0.495	0.091	0.063	0.083	1.889	21.24	2.036	0.211	0.097	0.235	0.666	6.044	16.6
	fine	rectangular	3.224	1.092	0.584	0.382	1.647	19.58	7.931	4.448	2.896	11.9	3.261	1.524	0.89	10.69	3.122	1.581	1.003	4.381	1.941	1.884	1.842	3.423	0.516	0.185	0.11	1.177	49.6	7.834	0.703	0.1	0.123	1.938	9.206	1.645
		equilateral	3.561	1.216	0.651	0.427	1.654	17.41	6.968	3.88	2.517	9.99	2.714	1.265	0.741	9.244	2.648	1.314	0.823	5.132	2.007	1.919	1.901	4.197	0.625	0.223	0.126	1.22	43.21	8.513	0.758	0.106	0.125	2.066	7.798	1.654
		randomized	3.618	1.206	0.654	0.431	2.008	19.99	7.763	4.265	2.745	12.65	3.692	1.759	1.037	10.22	2.848	1.476	0.97	4.938	1.987	1.917	1.899	4.042	0.605	0.215	0.123	2.085	51.82	8.531	0.765	0.106	0.119	2.055	11.72	1.819
		random	4.387	2.329	2.392	2.675	10.2	54.32	19.9	11.41	8	84.7	45.68	30.93	21.78	384.2	193	81.83	261.8	4.577	1.964	1.895	1.876	3.922	0.605	0.233	0.148	12.98	137.6	7.854	0.764	0.294	0.39	1.903	40.63	6.557
implicit exact	coarse	MC	0.599	1.044	1.443	1.693	364.8	2.041	1.005	0.849	0.929	5.143	1.675	1.226	1.323	26.77	8.416	6.086	9.234	1.933	1.885	1.885	1.916	0.446	0.789	1.126	1.454	0.866	4.634	0.97	0.605	1.298	1.9	1.967	1.7	8.377
	medium		0.696	0.565	0.68	0.823	168	6.355	2.892	1.935	1.494	25.76	8.883	4.467	2.792	141.4	50.29	20.15	12.88	2.158	1.951	1.956	1.973	0.517	0.272	0.401	0.571	1.761	14.4	1.286	0.445	1.005	1.856	2.062	5.132	21.88
	fine		2.552	1.482	1.218	1.094	66.37	26.47	11.76	7.889	6.148	136.1	71.27	47.27	33.53	774.6	240.8	210.4	120.7	2.651	1.654	1.57	1.567	1.668	0.311	0.159	0.158	7.213	62.32	3.643	0.363	0.279	0.754	2.184	22.64	23.96
	coarse	DC	0.373	0.674	1.012	1.196	5.134	0.677	0.373	0.406	0.544	0.531	0.472	0.637	0.831	1.026	1.223	2.365	2.146	1.591	1.511	1.509	1.528	0.303	0.484	0.742	1.019	0.8	1.925	1.583	0.358	0.581	1.232	1.527	0.471	0.95
	medium		0.396	0.314	0.396	0.508	14.3	1.889	0.729	0.425	0.32	1.181	0.402	0.3	0.317	1.013	0.36	0.592	0.809	1.952	1.73	1.707	1.704	0.409	0.151	0.219	0.337	0.335	5.685	2.069	0.295	0.412	0.962	1.397	1.097	3.147
	fine		1.453	0.936	0.723	0.643	50.3	7.442	2.735	1.474	0.954	4.475	1.245	0.603	0.4	3.757	1.037	0.531	0.395	4.403	3.888	3.839	3.801	1.517	0.242	0.114	0.152	0.908	24.26	4.359	0.433	0.433	1.17	3.04	4.327	13.01
implicit inexact	coarse	MC	0.606	1.063	1.47	1.725	331	2.423	1.117	0.912	0.985	5.109	1.628	1.235	1.351	25.82	8.246	7.331	4.83	1.945	1.907	1.913	1.946	0.437	0.807	1.158	1.488	0.93	4.892	0.986	0.613	1.324	1.651	1.999	1.768	9.119
	medium		0.696	0.562	0.676	0.819	168.3	6.598	3.033	2.025	1.549	25.16	8.731	4.402	2.662	164.7	612.8	17	17.18	2.165	1.95	1.958	1.975	0.519	0.27	0.401	0.573	1.784	14.51	1.281	0.446	1.009	1.55	2.066	5.167	21.99
	fine		2.542	1.506	1.252	1.135	66.23	26.6	11.91	8.006	6.239	136.5	72.81	48.65	33.62	551.1	239.7	126.6	107.3	2.653	1.657	1.569	1.567	1.666	0.312	0.16	0.159	7.139	62.33	3.624	0.36	0.277	0.585	2.176	22.6	23.88
	coarse	DC	0.42	0.692	0.929	1.085	5.573	0.748	0.415	0.418	0.53	0.564	0.461	0.603	0.777	0.704	1.481	1.428	1.673	1.611	1.523	1.505	1.513	0.363	0.514	0.716	0.967	0.849	2.116	1.63	0.43	0.7	0.843	1.484	0.54	1.078
	medium		0.402	0.323	0.401	0.513	14.47	1.909	0.737	0.432	0.326	1.193	0.407	0.303	0.318	1	0.365	0.699	0.938	1.953	1.732	1.707	1.703	0.417	0.156	0.222	0.339	0.339	5.779	2.086	0.301	0.414	0.858	1.396	1.122	3.199
	fine		1.444	0.927	0.719	0.638	49.72	7.363	2.71	1.462	0.946	4.449	1.233	0.599	0.395	3.736	0.994	0.527	0.389	4.407	3.896	3.839	3.801	1.521	0.241	0.115	0.154	0.898	24.01	4.314	0.429	0.437	1.039	3.029	4.312	12.83
nurbs	coarse	rectangular	0.422	0.174	0.174	0.223	15.77	2.823	1.178	0.662	0.434	1.696	0.501	0.275	0.234	1.517	0.467	0.249	0.189	1.513	1.233	1.206	1.195	0.442	0.096	0.142	0.28	0.173	6.73	5.457	0.755	0.167	0.263	0.793	1.254	3.452
		equilateral	0.451	0.174	0.154	0.186	16.36	2.922	1.203	0.676	0.442	1.74	0.504	0.264	0.198	1.645	0.484	0.25	0.179	1.542	1.2	1.176	1.172	0.488	0.093	0.123	0.253	0.177	6.931	5.813	0.778	0.161	0.23	0.848	1.296	3.689
		randomized	0.463	0.177	0.166	0.2	20.56	3.318	1.335	0.743	0.484	2.159	0.662	0.35	0.254	1.823	0.529	0.291	0.215	1.519	1.175	1.162	1.159	0.497	0.091	0.123	0.255	0.314	8.405	5.863	0.801	0.166	0.23	0.836	1.886	4.699
		random	0.791	0.72	1.041	1.224	72.51	10.12	3.845	2.219	1.577	15.26	5.886	3.12	2.19	28.14	87.65	252.6	19.27	1.64	1.344	1.302	1.296	0.515	0.182	0.272	0.429	2.143	26.01	6.012	0.848	0.218	0.368	0.832	7.371	19.11
	medium	rectangular	0.841	0.297	0.191	0.174	35.64	6.481	2.662	1.496	0.973	3.819	1.072	0.525	0.338	3.421	1.017	0.521	0.335	2.064	1.533	1.47	1.436	0.84	0.14	0.111	0.201	0.352	15.46	4.685	0.493	0.175	0.396	1.048	2.858	8.489
		equilateral	0.932	0.323	0.196	0.164	36.47	6.536	2.661	1.495	0.973	3.871	1.082	0.524	0.33	3.667	1.058	0.525	0.332	2.16	1.52	1.443	1.41	0.969	0.157	0.108	0.188	0.379	15.7	5.029	0.52	0.162	0.362	1.072	2.908	8.95
		randomized	0.932	0.323	0.201	0.172	44.51	7.482	2.958	1.64	1.06	4.786	1.431	0.699	0.44	4.002	1.12	0.582	0.391	2.12	1.505	1.432	1.401	0.951	0.149	0.107	0.191	0.666	19.01	5.041	0.525	0.167	0.363	1.067	4.35	10.95
		random	1.485	1.188	1.464	1.61	81.4	20.77	7.675	4.363	3.058	27.64	11.49	7.252	4.779	109.2	26.57	27.43	10.11	2.135	1.593	1.524	1.5	0.97	0.21	0.203	0.301	4.295	51.8	4.835	0.536	0.252	0.494	1.041	14.84	27.21
	fine	rectangular	2.164	0.731	0.403	0.273	24.51	17.66	7.224	4.037	2.616	10.31	2.842	1.339	0.792	9.231	2.714	1.375	0.873	3.322	1.858	1.711	1.667	2.146	0.328	0.144	0.146	0.935	41.66	4.608	0.44	0.243	0.519	1.215	7.953	7.698
		equilateral	2.373	0.799	0.439	0.295	19.34	17.57	7.055	3.934	2.551	10.28	2.826	1.329	0.783	9.741	2.76	1.353	0.833	3.583	1.865	1.686	1.639	2.431	0.37	0.155	0.148	0.964	42.16	4.914	0.46	0.229	0.489	1.25	7.936	6.342
		randomized	2.403	0.799	0.441	0.3	22.29	20.02	7.825	4.305	2.771	12.68	3.744	1.794	1.062	10.74	2.949	1.497	0.963	3.495	1.849	1.679	1.633	2.427	0.359	0.152	0.148	1.775	50.69	4.9	0.459	0.226	0.48	1.235	11.92	8.369
		random	3.868	2.562	2.607	2.68	35.75	55.38	21.23	12.46	8.91	80.12	43.02	30.4	21.81	331.5	133	86.84	48.13	3.414	1.909	1.768	1.738	2.493	0.415	0.233	0.231	12.24	132.9	4.602	0.497	0.299	0.561	1.207	38.89	17.69

Results


data
sampling
triangulation
explicit functions
implicit functions
NURBS surfaces
explicit functions
implicit functions
NURBS surfaces
estimators
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noisy data

Results

- all results are in the paper and supplementary material
- results can be reproduced using our software

- general observations:
  - no single winner
  - big differences with noise
  - big differences with sampling

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Meta-estimator

- analyzes the input mesh
- chooses appropriate estimator

Statistics

- capture significant global characteristics of meshes
- computed locally
  - per vertex
  - per triangle
  - per edge
  - per corner
- pooling operators
  - minimum/maximum
  - median, mean
  - standard deviation

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Statistics

- edge lengths (relative to average)
- dihedral angles (signed, unsigned)
- triangle inner angles
- triangle circumcircle/inscibed circle ratio
- vertex adjacent solid angles
- vertex degrees
- differential coordinates
  - uniform
  - cotan discretization
  - mean value discretization

Laplacian discretizations

Uniform discretization of Laplacian

$$\mathbf{d}_i^u = \frac{1}{\|N(i)\|} \sum_{j \in N(i)} (\mathbf{v}_j - \mathbf{v}_i)$$

captures sampling irregularity and normal offset

Mean value Laplacian

$$\mathbf{d}_i^m = \sum_{j \in N(i)} \frac{w_{ij}(\mathbf{v}_j - \mathbf{v}_i)}{\sum_{j \in N(i)} w_{ij}}, w_{ij} = \frac{tan(\alpha/2) + tan(\beta/2)}{\|\mathbf{v}_i - \mathbf{v}_j\|}$$

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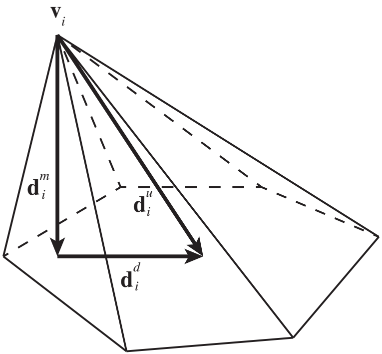
captures normal offset

Laplacian difference

Laplacian difference

$$\mathbf{d}_i^d = \mathbf{d}_i^u - \mathbf{d}_i^m$$

captures sampling irregularity



Smoothness statistic

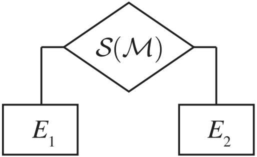
- uses cotan discretization of mesh Laplacian L
- measures smoothness of the mean curvature normal vectors:

$$\mathbf{s} = \bar{l}^2 L^2 \mathbf{v}$$

- scaling by squared average edge length $\bar{l}$ ensures scale independence
- per-vertex vector lengths used as input to pooling operators

Meta-estimator results

- evaluate a statistic, decide between two estimators
- tested all combinations
- threshold learned from a subset of 35% of the meshes



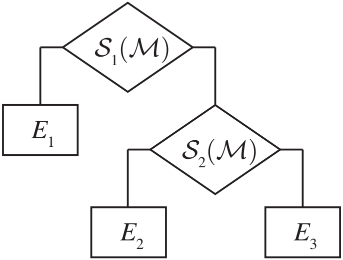
Meta-estimator results

- evaluate a statistic, decide between two estimators
- tested all combinations
- threshold learned from a subset of 35% of the meshes

- best results:
  - median s used for the desicison
  - fitting circles [Goldfeather and Interrante] ($\bar{e} = 15.19$)
  - generalized shape operator [Hildebrandt and Polthier] ($\bar{e} = 37.99$)
  - result: $\bar{e} = 1.18$

2-level meta-estimator

- threshold for $\mathcal{S}_2$ determined first
- $E_2 + \mathcal{S}_2 + E_3$ treated as a single estimator in order to determine the threshold for $\mathcal{S}_1$
- computationally expensive
- optimal values not guaranteed



2-level meta-estimator

- threshold for $\mathcal{S}_2$ determined first
- $E_2 + \mathcal{S}_2 + E_3$ treated as a single estimator in order to determine the threshold for $\mathcal{S}_1$
- computationally expensive
- optimal values not guaranteed

- best results:
  - median s used as both $\mathcal{S}_1$ and $\mathcal{S}_2$ (different thresholds)
  - cotan Laplacian [Meyer] as E_1
  - generalized shape operator [Hildebrandt and Polthier] as E_2
  - adaptive II estimation [Kalogerakis et al.] as E_3
  - $\bar{e} = 0.86$

Experiments with noiseless data only

- single level meta-estimator:
  - mean of Laplacian difference $\mathbf{d}^d$ used as threshold (captures regularity of sampling)
  - decision between fitting circles [Goldfeather and Interrante] and cotan Laplacian [Meyer]
  - resulting $\bar{e} = 0.45$
- 2-level meta-estimator
  - top-level choice based on mean Laplacian difference $\mathbf{d}^d$
  - bottom-level choice based on std. deviation of mean value Laplacian vector lengths
  - cotan Laplacian [Meyer] as E_1
  - circle fitting [Goldfeather and Interrante] as E_2 and E_3 (different radius)
  - $\bar{e} = 0.31$

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Conclusions

- results confirm intuitive common knowledge on curvature estimation
- the experiment is very easy to modify
- new estimators can be easily added and tested
- available online: http://graphics.zcu.cz/curvature.html

future work

- neural networks for meta-estimators
- localized meta-estimators
- test normal estimators
- test principal curvature direction estimators

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Questions?

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